How do shapes fill space?

What shapes fill the plane?

Factsheet 6

A History of Aperiodicity

In the early 1960's the mathematician Wang was thinking about the question:

If you are given a set of shapes of tiles, can you use them to tile the whole plane?

Wang thought that there should be an algorithm or computer program that would be able to decide, for any set of shapes of tiles you gave it, whether or not you could use them to fill the plane. He was assuming however that if you could fill the plane with your tiles, then it would have to be by using them to form a patch that could be endlessly repeated, just as we saw on the poster with the Myers tile.

If it had been true that any tiling of the plane had to be the repetion of some basic patch, then Wang's algorithm would have gone as follows:

- Take one tile shape. Does it form a patch that can be made to repeat?
  If it does, you've got your answer. If not, test each other shape.
- If all the shapes fail, try all possibilities of 2 shapes put together.
  If one of them forms a patch that repeats then you're finished.
  If not, test all ways of putting 3 shapes together, and so on.

Either – you eventually get to a patch that can be made to repeat:
you can tile the plane with these shapes of tiles;
Or – you get to a point where you can't fit any more tiles together:
you can't tile the plane with these shapes of tiles.

However, in 1966 Wang's student Berger showed there was a problem with this: he discovered a set of tiles that did tile the plane, but in a way that never repeated. In fact, the tiles could never be put together to form a patch that could be repeated: the tiles have to be continually put together in different configurations. Any algorithm of the sort outlined above would never finish. Berger had found the first example of an aperiodic tiling in 1966. He needed 20,426 different tiles to make it. Once the first example had been found, others quickly found aperiodic tilings using smaller sets of tiles. By 1971 Raphael Robinson had a tiling using just 6 tiles, but a few years later Roger Penrose had his, using just 2 tiles.

Less than 10 years later the patterns these tiles made had been found in nature as the positions of atoms in metal alloys. The 3 dimensional analogues of these 2-d tilings are now understood as key shapes in the structure of viruses and are being used to understand and predict virus evolution.

The quest for a monotile. Penrose's tiling is made of just two shapes of tile. At the moment, no one knows if there is a single shape of tile that tiles the plane but only aperiodically. Can you find one, or prove that there can be no such tile?

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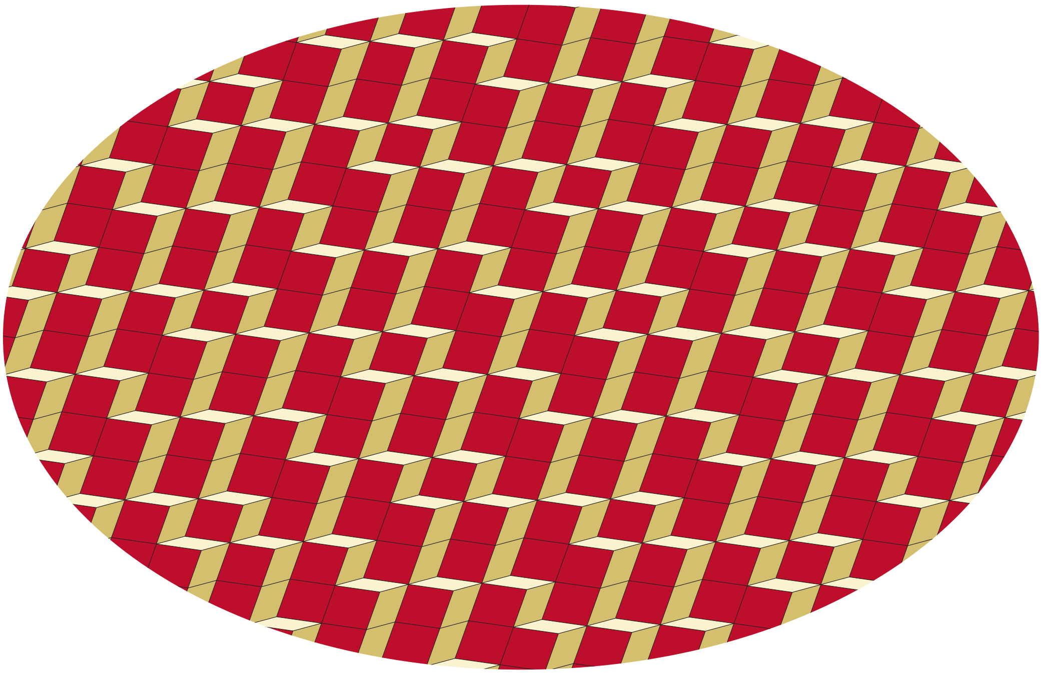
Non-periodic slices

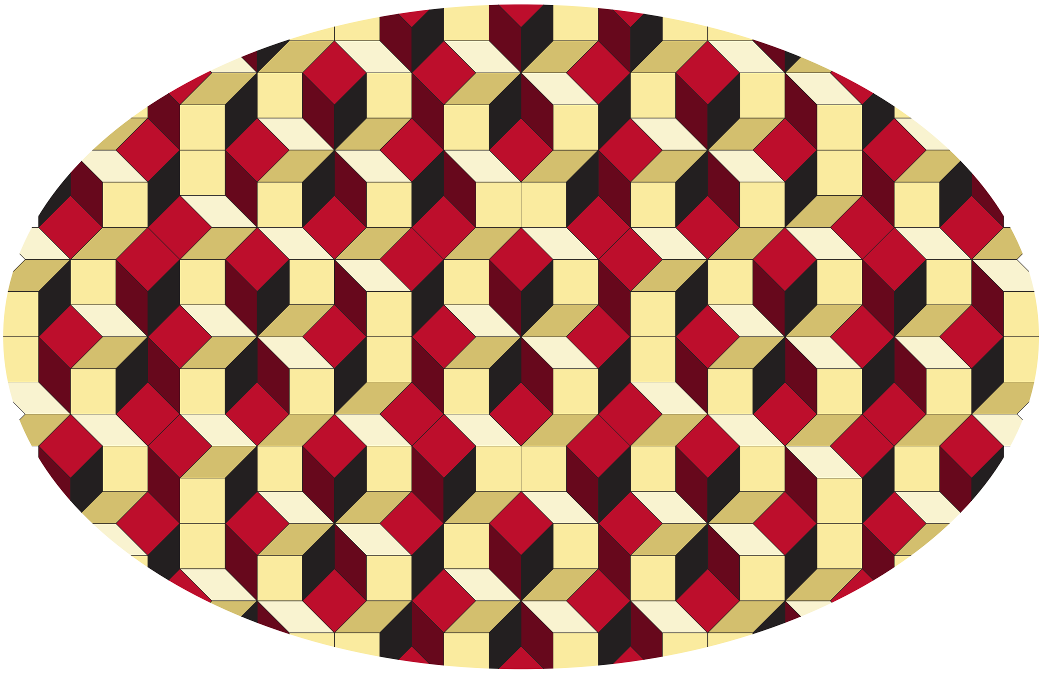
How can we easily generate non-periodic tilings? Here is a method that has proved to be very useful. Take a look at this:



Is this a picture of a stack of blocks, or is it a tiling made from 3 different shapes of tile?

Well it's both! One way to create unlimited examples of aperiodic tilings is to take a periodic tiling of a higher dimensional space - the picture above used a tiling of 3d space by cubes - and then taking a slice through it. So long as the slice is at an irrational slope with respect to the sides of the cubes the result is a non-periodic pattern of tiles.

Here is another example:



this time a 2d slice through a tessellation of 4d by 4 dimensional cubes. The Penrose tiling can be made as a slice through 5d space.

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