SELECTED 2009 THE ROYAL SOCIETY SUMMER SCIENCE EXHIBITION

New Maths, New Science!

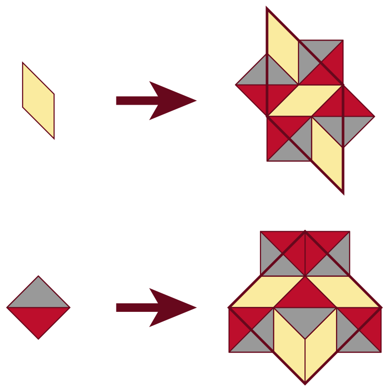
Factsheet 7

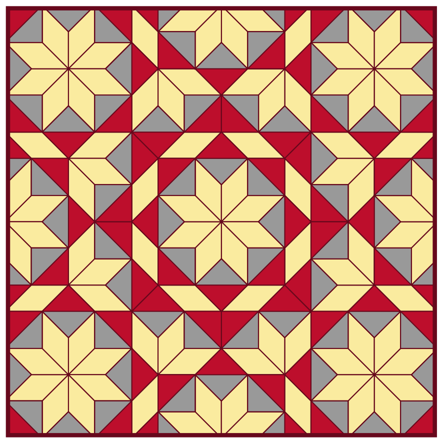
Tilings Zoo

We now have a large zoo of aperiodic tilings, though there are still very few simple ones. New examples with less than even ten tiles that are not related to old examples are still of great interest to mathematicians. A powerful of finding aperiodic tilings was given by Goodman-Strauss in 1997. He proved that the tiles for any substitution tiling (see Factsheet 5) could be turned into an aperiodic set by adding jigsaw like notches to the edges and corners. Unfortunately in most cases this also vastly increased the number of tiles required. Aperiodicity is therefore still a very mysterious phenomenon. Here are some of the many substitution tilings that can be made into aperiodic tilings. For more examples see the *Tilings Encyclopedia*: http://tilings.math.uni-bielefeld.de/

Ammann-Beenker Tiling

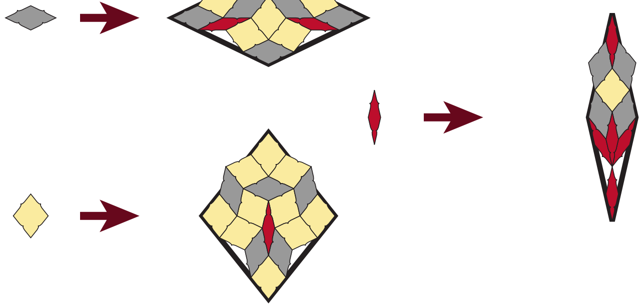
This tiling can be constructed both as a substitution tiling, and as a slice through 4 dimensional space filled with 4d cubes. It is therefore related to the Penrose tiling, which can be made as a substitution tiling and as a slice of a periodic tiling of 5d space. It was first constructed in 1977 by amateur mathematician Robert Ammann, a US postal worker.

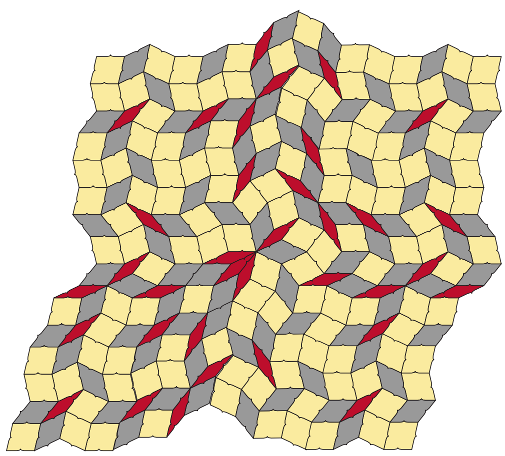




Goodman-Strauss 7-fold rhomb Tiling

This beautiful tiling was discovered by Chaim Goodman-Strauss. It is part of a family of rhomb substitution tilings that have n-fold rotational symmetry for any n, discovered by Edmund Harriss in 2004.

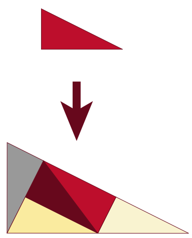


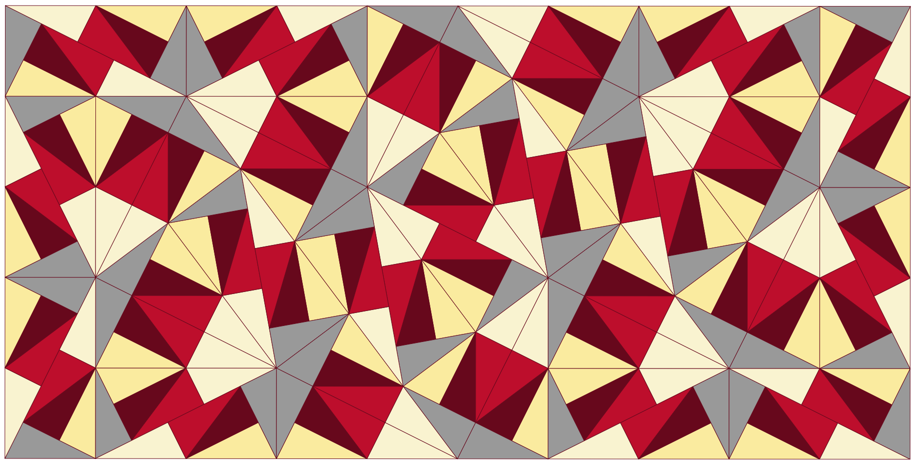


Edmund Harriss, Uwe Grimm, Richard Henry, John Hunton, Vinay Kathotia, Jeroen Lamb

Pinwheel Tiling

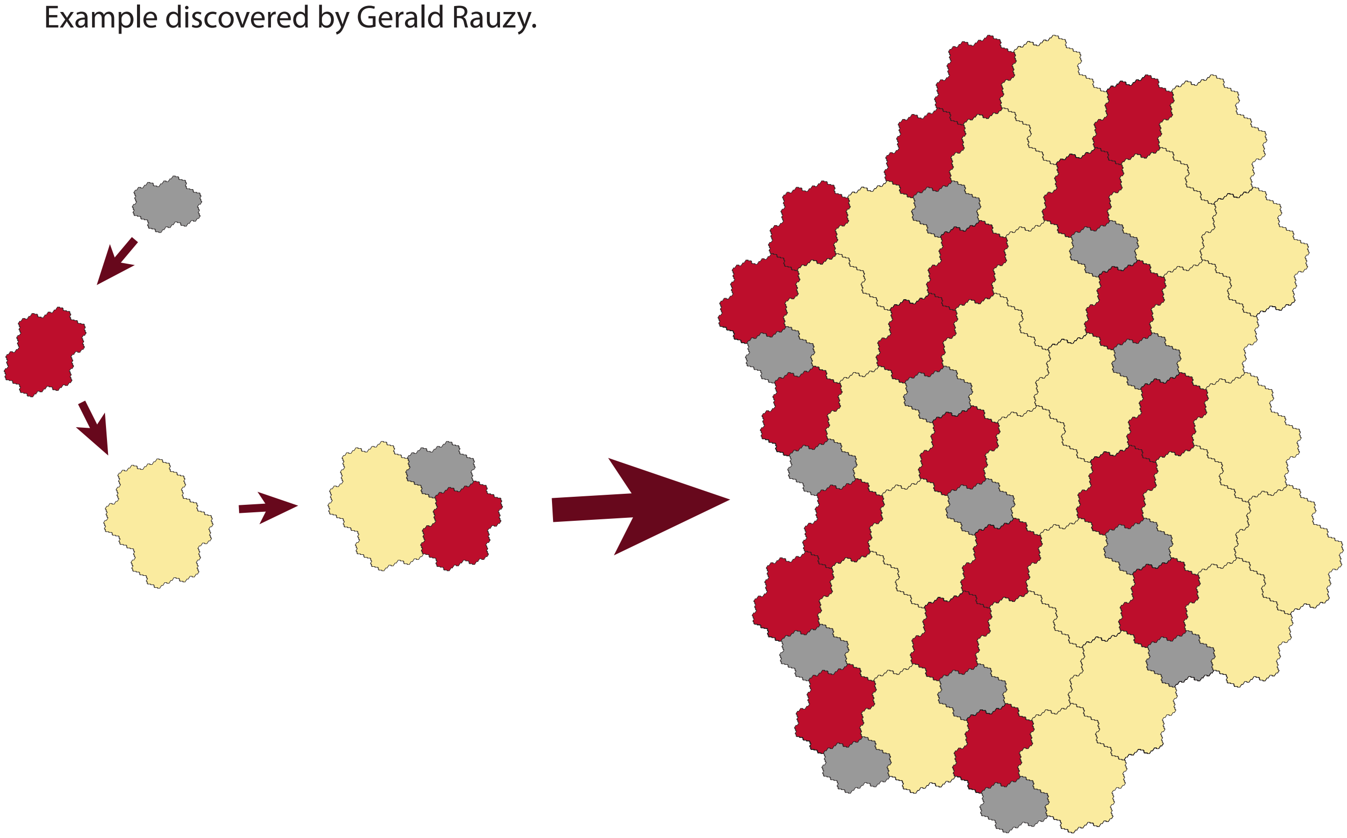
Pinwheel: The Pinwheel tiling comes from a simple substitution rule on a single right angled triangle with side lengths 1, 2 and √5. Although it has only one shape of tile, this tile appears in an infinite number of rotations in the whole tiling. This complexity makes it hard to analyse and it is only in research carried out in the last year that we have begun to fully understand its properties. Nevertheless, it has already been used in architecture; the Federation Square buildings in Melbourne, Australia are built and decorated with a giant version of this tiling.





Rauzy Tiling

In the pinwheel tiling and our original example of a substitution tiling, the chair tiling on Factsheet 5, the expanded tile was exactly divided into the new tiles. In the other example this is not the case. We can always change into a version of the tiling where the expanded tiles are divided. In most cases this is at the cost of having fractal boundaries, not straight lines, as shown in this Example discovered by Gerald Rauzy.



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Edmund Harriss, Uwe Grimm, Richard Henry, John Hunton, Vinay Kathotia, Jeroen Lamb